GRAVITATIONAL WAVES in RELATIVISTIC THEORY of GRAVITATION

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Abstract

It is shown that gravitational waves do not have nonphysical "ghost" states in the Relativistic Theory of Gravitation with graviton having nonzero rest mass due to the causality condition.

It was shown in [1,2] that, in linearized theory of gravitation, introducing the rest mass of graviton for a field with spins 2 and 0 leads to nonphysical "ghost states" due to spin 0 when interpreting gravitational effects in the Solar system. "Ghost" states appear also in the gravitational radiation. A conviction has grown up on this basis that just this proves that the graviton mass is exactly zero. However, in their study the authors of [1,2] do not treat the gravitational field as a tensor **physical field** in Minkowski space which generates an effective Riemannian space, and as a consequence, its own causality cone. That is why the **causality condition** has not appeared whereas it arises in the Relativistic Theory of Gravitation (RTG) [3], because it has two causality cones. There is only one causality cone of the Riemannian space in General Relativity.

In the study of gravitational radiation it was found in paper [4] that taking into account nonlinear terms allows one to eliminate "ghost" states. This treatment was admitted also by us and it was given in [3]. In this work we have obtained that, in RTG, the negative energy flow and, hence, "ghost" states are eliminated even without an account of nonlinear terms. It turns

out to be¡sufficient to fulfill the causality condition from the RTG. Following papers [5,6,7] We start from the opportunity of existence of a free gravitational field – the gravitational waves, as an objective physical reality similar to electromagnetic waves in vacuum.

For the simplicity and accuracy of our analysis we consider a weak plane gravitational wave in vacuum with amplitude $a^{\mu\nu}(k)$, propagating along Z axis

$$\Phi^{\mu\nu} = a^{\mu\nu}(k) \cos kx, \tag{1}$$

where $k_\nu = (\omega, 0, 0, -q\omega)$, $q^2 = 1 - \frac{m^2}{\omega^2}$, and m is the graviton mass.

We use system of units conventions $G = \hbar = c = 1$. In vacuum the basic RTG equations in linear approximation and in an inertial frame with Galilean coordinates are taking the following form

$$\Box\, \Phi^{\mu\nu} + m^2 \Phi^{\mu\nu} = 0, \tag{2}$$

$$\partial_\nu \Phi^{\mu\nu} = 0. \tag{3}$$

The wave (1) is a solution of these equations. A weak gravitational field $\Phi^{\mu\nu}$ produces an effective Riemannian space with the following metric tensor

$$g_{\mu\nu} = \gamma_{\mu\nu} - \Phi_{\mu\nu} + \frac{1}{2}\gamma_{\mu\nu}\Phi, \quad \Phi_{\mu\nu}\gamma^{\mu\nu} = \Phi;$$

tensor $g^{\mu\nu}$ is given by the analogous expression

$$g^{\mu\nu} = \gamma^{\mu\nu} + \Phi^{\mu\nu} - \frac{1}{2}\gamma^{\mu\nu}\Phi. \tag{4}$$

It follows from the above that scalar curvature of the effective Riemannian space R is

$$R = \frac{1}{2} m^2 \Phi.$$

But it occurs so that it does not influence the energy flow, as we shall see below. Minkowski space interval in an inertial frame with Galilean coordinates is

$$d\sigma^2 = \gamma_{\mu\nu} dx^\mu dx^\nu = dt^2 - dx^2 - dy^2 - dz^2. \tag{5}$$

As RTG treats the gravitational field as a physical tensor field propagating in Minkowski space, the causality cone of the arising effective Riemannian space should not go out the causality cone of the Minkowski space. Just this is the causality principle of the RTG. According to this principle, the timelike and lightlike geodesics of the effective Riemannian space which is produced by the physical field should not go outside boundaries of the Minkowski space cone. Just this physical requirement should get the proper mathematical formulation.

The terms with second derivatives over spacetime coordinates appear in the **hyperbolic dynamical equations** of the gravitational field in RTG in the following form

$$g^{\mu\nu} \, \frac{\partial^2 \Phi^{\alpha\beta}}{\partial x^\mu \partial x^\nu} \, . \tag{6}$$

The characteristic equation for the gravitational equations is provided by higher order derivative terms (6) only

$$g^{\mu\nu} \, \frac{\partial S}{\partial x^\mu} \, \frac{\partial S}{\partial x^\nu} = 0. \tag{7}$$

This equation determines wavefront of the field, if graviton have no rest mass. **The characteristics determine the causality cone of effective Riemannian space.** Each term with a second derivative from (6) has the corresponding term in characteristics (7). If some term with a second derivative is absent in (6), then there will be no corresponding term in (7).

The timelike geodesic lines in correspondence with (7) are given by the Hamilton-Jacobi equations

$$g^{\mu\nu}\frac{\partial S}{\partial x^\mu}\,\frac{\partial S}{\partial x^\nu} = 1. \tag{8}$$

The total set of geodesic lines in correspondence with (6) is determined by the following equations

$$g^{\mu\nu}\frac{\partial S}{\partial x^\mu}\,\frac{\partial S}{\partial x^\nu} = \left\{ \begin{array}{r} 0 \\ 1 \\ -1 \end{array} \right. ,$$

where the first equation gives the isotropic geodesics, the second – timelike geodesics, whereas the third gives spacelike geodesic lines.

Therefore, on the base of Eqs. (7) and (8) isotropic and timelike geodesic lines, in correspondence with Eq. (6), fulfill the following inequality

$$g^{\mu\nu}\frac{\partial S}{\partial x^\mu}\,\frac{\partial S}{\partial x^\nu} \geq 0. \tag{9}$$

This inequality may be written as follows

$$g_{\alpha\beta}p^\alpha p^\beta \geq 0, \tag{10}$$

where contravariant vector p^α is

$$p^\alpha = g^{\alpha\mu}\frac{\partial S}{\partial x^\mu}. \tag{11}$$

To provide that the causality cone of the effective Riemannian space be inside the causality cone of Minkowski space it is necessary and sufficient to fulfill the following inequality

$$\gamma_{\alpha\beta}p^\alpha p^\beta \geq 0. \tag{12}$$

Inequalities (10) and (12) may be written in a form directly connected with the geodesic motions (7) and (8) which are in exact correspondence with (6)

$$g^{\mu\nu} p_\mu p_\nu \geq 0, \tag{13}$$

$$\gamma_{\alpha\beta} g^{\alpha\mu} g^{\beta\nu} p_\mu p_\nu \geq 0, \tag{14}$$

where covariant vector p_ν is

$$p_\nu = \frac{\partial S}{\partial x^\nu}. \tag{15}$$

Causality conditions (13) and (14) put definite rigid restrictions on solutions of the gravitational field equations. Only the solutions satisfying inequalities (13) and (14) have a physical meaning in the theory. Inequalities (13) and (14) **are straightforwardly connected with the hyperbolic equations for the gravitational field** as they are derived from **the second derivatives structure (6)** of the dynamical equations. Just this mathematical formulation of the causality principle guarantees the position of the Riemannian causality cone inside the causality cone of the Minkowski space, in correspondence with the dynamical structure (6).

Earlier in [3] we have not recognized this fact of necessity to establish the direct correspondence of the causality principle with the **hyperbolic dynamical system of equations** . In the case of static system the gravitational field equations are not hyperbolic and so such a direct correspondence is absent. But in that case the causality condition can be used in the form of inequalities (10) and (12). Taking into account

$$\tilde{g}^{\mu\nu} = \tilde{\gamma}^{\mu\nu} + \tilde{\Phi}^{\mu\nu},$$

where

$$\tilde{g}^{\mu\nu} = \sqrt{-g} g^{\mu\nu}, \;\; \tilde{\gamma}^{\mu\nu} = \sqrt{-\gamma}\gamma^{\mu\nu}, \;\; \tilde{\Phi}^{\mu\nu} = \sqrt{-\gamma}\Phi^{\mu\nu},$$

inequalities (13) and (14) in inertial frame with Galilean coordinates take the following form

$$(\gamma^{\mu\nu} + \Phi^{\mu\nu}) p_\mu p_\nu \geq 0, \tag{16}$$

$$\gamma_{\alpha\beta}(\gamma^{\alpha\mu} + \Phi^{\alpha\mu})\,(\gamma^{\beta\nu} + \Phi^{\beta\nu}) p_\mu p_\nu \geq 0. \tag{17}$$

For the motion (1) the following characteristic equation is valid

$$g^{\mu\nu} p_\mu p_\nu = g^{00}(p_0)^2 + 2g^{03} p_0 p_3 + g^{33}(p_3)^2 = 0.$$

For the weak gravitational field and **the special motion (1) along Z-axis** inequality (16) is fulfilled if the value of x defined as

$$x = \frac{p_3}{p_0},$$

is limited by the following inequalities

$$x_1 \leq x \leq x_2, \tag{18}$$

where

$$\begin{aligned} x_1 &= \Phi^{03} - 1 - \frac{1}{2}(\Phi^{00} + \Phi^{33}), \\ x_2 &= \Phi^{03} + 1 + \frac{1}{2}(\Phi^{00} + \Phi^{33}). \end{aligned} \tag{19}$$

So we have defined the set of timelike vectors laying inside the causality cone determined by characteristics on the base of (6) for the motion (1), leading to metric (4). Inequality (17) will be fulfilled if

$$x_1' \leq x \leq x_2',$$

where

$$x_1' = 2\Phi^{03} - 1 - \Phi^{00} - \Phi^{33},$$
$$x_2' = 2\Phi^{03} + 1 + \Phi^{00} + \Phi^{33}. \tag{20}$$

In order to provide the position of the effective Riemannian causality cone inside the Minkowski causality cone it is necessary and sufficient to fulfill the following inequalities

$$x_1' \leq x_1, \quad x_2 \leq x_2'. \tag{21}$$

On the base of (21) and taking into account (19) and (20) we obtain

$$\Phi^{00} \pm 2\Phi^{03} + \Phi^{33} \geq 0. \tag{22}$$

From Eq. (3) we find for the solution (1) :

$$\Phi^{00} = q\Phi^{03}, \quad \Phi^{03} = q\Phi^{33}. \tag{23}$$

After substituting these equations into (22) we obtain

$$(q \pm 1)^2 \Phi^{03} \geq 0. \tag{24}$$

It follows from these inequalities for the wave (1) that

$$\Phi^{03} \equiv 0, \tag{25}$$

and therefore, on the base of Eqs. (23), the following equations take place

$$\Phi^{00} \equiv 0, \quad \Phi^{33} \equiv 0. \tag{26}$$

In RTG the causality principle selects the physical solution of the gravitational equations. It follows from (25) and (26) that

longitudinal-longitudinal components are absent in the wave solution (1). Just for this reason there are no term like

$$R\Phi^{03} = \frac{1}{2}m^2\Phi\Phi^{03},$$

in the energy flow, this term is identically zero.When the graviton mass is zero, Eqs. (25),(26) as a rule are derived from the gauge transformations. Here they follow from the causality principle. Just this provides the positivity of the energy flow in RTG in case of the nonzero graviton mass.

In the RTG quadratic approximation considered in Galilean coordinates the energy flow is determined, according to [3,4], by means of the **tensor** quantity

$$t_g^{\epsilon\lambda} = \frac{1}{32\pi}\gamma^{\epsilon\alpha}\gamma^{\lambda\beta}\left(\partial_\alpha\Phi_\nu^\tau\cdot\partial_\beta\Phi_\tau^\nu - \frac{1}{2}\partial_\alpha\Phi\cdot\partial_\beta\Phi\right). \qquad (27)$$

Rising and lowering of the indices for $\Phi^{\mu\nu}$ is provided by means of metric tensor $\gamma_{\mu\nu}$. According to Eq. (3), the following relations take place for solution (1):

$$\begin{aligned} a^{10} &= qa^{13} \\ a^{20} &= qa^{23}. \end{aligned} \qquad (28)$$

Taking into account (1), and also Eqs. (25), (26) and (28), on the base of Eq. (27) we obtain for the wave (1) after averaging over time

$$t_g^{03} = \frac{1}{32\pi}q\omega^2\left\{(a_1^2)^2 + \frac{1}{4}(a_1^1 - a_2^2)^2 + \frac{m^2}{\omega^2}\left[(a_3^1)^2 + (a_3^2)^2\right]\right\}. \qquad (29)$$

It follows from here that only transverse-transverse components are present in the density of the flow for the wave (1), and

longitudinal-transverse a_3^1, a_3^2, a_0^1, a_0^2 also. The last ones are multiplied by $\frac{m^2}{\omega^2}$ in the energy flow (29).The longitudinal-longitudinal components **are absent** in the wave (1). It should be noted that according to RTG it is possible to provide a continuous transformation to the zero graviton mass in this problem.

Therefore it follows from (29) that the presence of nonzero graviton mass does not lead in the RTG to the appearance of nonphysical "ghost" states. The "ghost" states also do not appear in the RTG when explaining the Solar system effects. Here there is a continuous transformation to the zero graviton mass at the distance from the source $r \gg r_g = 2M$. The graviton mass arises in the RTG with the necessity when we begin to treat the gravitational field as a physical one in Minkowski space.

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